

John W. Severinghaus. Simple, accurate equations for human blood O₂ dissociation computations. *J. Appl. Physiol.: Respirat. Environ. Exercise Physiol.* 46(3):599-602, 1979. revisions, 1999, 2002, 2007

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Abstract:

Hill's equation can be modified by adding a cubic term to match the standard human blood O₂ dissociation curve to within ±0.0055 S (mean error=0.0026) where S = functional saturation from 0<S<1 (Eq. 1). Inversion (by R. K. Ellis) permits PO₂ to be computed from S (Eq. 2). Another modification of Hill's equation may be used to estimate the temperature coefficient f_T of PO₂ which varies with the range of PO₂ (Eq. 3). The conventional Bohr coefficient (-1.1), the effect of pH on lnPO₂ at constant saturation, varies with S (or PO₂). A corrected Bohr factor may be computed from Eq. 4.

$$S = ((23,400((PO_2)^3 + 150PO_2)^{-1}) + 1)^{-1} \quad (1)$$

$$PO_2 = (B+A)^{1/3} - (B-A)^{1/3} \quad (2)$$

$$\text{where } A = 11700(S^{-1}-1)^{-1} \text{ and } B = (50^3 + A^2)^{0.5}$$

$$f_T = \Delta \ln PO_2 / \Delta T = 0.058(0.243(PO_2/100)^{3.88} + 1)^{-1} + 0.013 \quad (3)$$

$$\Delta \ln PO_2 / \Delta pH = (PO_2/26.7)^{0.184} + 0.003 \text{ BE} - 2.2 \quad (4)$$

A method of computing P₅₀ from measured PO₂, pH, and S is presented.

Index terms: blood PO₂; Bohr effect; (a-v)O₂ content computation; O₂ saturation equation; pH effect on PO₂

THEORETIC ANALYSES OF O₂ EXCHANGES between gas, blood, and tissue and the effects of temperature and pH require reasonably accurate expressions of the O₂ dissociation curve and temperature coefficient. The Adair equation [7] although theoretically sound, could not be made to conform to whole blood O₂ dissociation to better than about ±1% saturation by Ed DeLand, using a Rand Corporation computer curve fitting program [7]. Both the Adair and several empiric power functions [1, 4, 5, 8, 10] use cumbersome multi-constant equations. To facilitate computation of the effect of heating skin on transcutaneous PO₂, I needed a simple accurate equation for the oxygen dissociation curve (ODC). By adding a second term with exponent 1.0 and varying the exponent "n" of the Hill equation, usually 2.7, a best fit was n = 3.0. This serendipitous equation suggested more than an empiric coincidence, possibly relating to the sudden increase in O₂ affinity of Hb as the 2nd O₂ is bound. However, there is no experimental or theoretical support for this relation as far as I know.

Definition of symbols and units of measurement.

PO ₂	partial pressure of O ₂ (Torr at 37°C)
S	O ₂ Hb/(O ₂ Hb + HHb) = 'functional saturation'/100
P ₅₀	PO ₂ of whole blood at 37°C, S = 0.5, pH = 7.4
C	O ₂ content (ml O ₂ STPD per ml blood)
BE	base excess of whole blood (meq/l)

I. O₂ Saturation from PO₂

Data (Table 1) used to describe the standard human blood O₂ dissociation curve at pH = 7.4, T = 37°C were compiled by Roughton and Severinghaus [7]. The Hill equation approximates the human blood O₂ dissociation curve reasonably well for S>0.3 but is unacceptably low at the bottom of the curve. The addition of a cubic term to Hill's equation corrected most of this misfit.

$$S = ((23,400((PO_2)^3 + 150PO_2)^{-1}) + 1)^{-1} \quad (1)$$

The greatest error is +0.55% at 98.77% Sat. The mean absolute error (with equal weight to each increment of saturation) is 0.26% saturation, less than half that of the Adair equation [7]. P₅₀ with Eq. 1 is 26.86, compared with 26.67 in the standard curve.

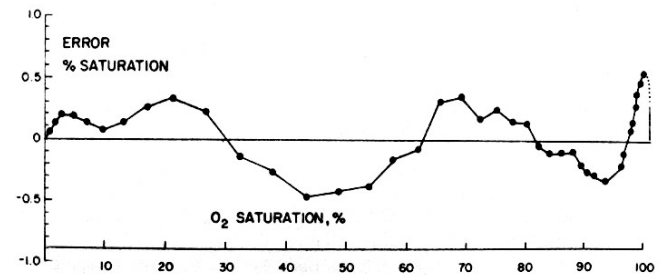


Fig 1. % saturation errors of equation 1 as function of S.

TABLE 1. Values for standard human blood O₂ dissociation curve at 37°C, pH = 7.4, extrapolated between data in [7].

PO ₂	%Sat	PO ₂	%Sat	PO ₂	%Sat
1	0.60	34	65.16	80	95.84
2	1.19	36	68.63	85	96.42
4	2.56	38	71.94	90	96.88
6	4.37	40	74.69	95	97.25
8	6.68	42	77.29	100	97.49
10	9.58	44	79.55	110	97.91
12	12.96	46	81.71	120	98.21
14	16.89	48	83.52	130	98.44
16	21.40	50	85.08	140	98.62
18	26.50	52	86.59	150	98.77
20	32.12	54	87.70	175	99.03
22	37.60	56	88.93	200	99.20
24	43.14	58	89.95	225	99.32
26	48.27	60	90.85	250	99.41
28	53.16	65	92.73	300	99.53
30	57.54	70	94.06	400	99.65
32	61.69	75	95.10	500	99.72

II. PO₂ from O₂ Saturation

Eq 1 was directly solved for PO₂ by Roger K. Ellis [2]. His solution may be simplified to:

$$PO_2 = (B+A)^{1/3} - (B-A)^{1/3} \quad (2)$$

where A = 11700(S⁻¹-1)⁻¹ and B = (50³ + A²)^{0.5}

III. Temperature Coefficient of PO₂ in Blood, f_T

The effect of changing temperature on blood PO₂ varies from 7.4%/°C at low saturation, to 1.3%/°C at high PO₂. This factor depends on the slope of the O₂ dissociation curve at the PO₂ of the blood. A modification of Hill's equation expresses slope, and may be used to compute the temperature coefficient at any given PO₂. The natural log of PO₂ rises per degree of warming at f_T:

$$f_T = \Delta \ln PO_2 / \Delta T = 0.058(0.243(PO_2/100)^{3.88} + 1)^{-1} + 0.013 \quad (3)$$

In Fig. 2, Eq. 3 is plotted as a function of both PO₂ and saturation %. This equation applies to PO₂ at 37°C. To begin with some other temperature, one may estimate a trial 37°C PO₂ using the factors 6%/°C if PO₂<100, and 6 Torr/°C above 100 Torr, and proceed iteratively with Eq. 3.

Little experimental data is available to verify the temperature correction. Nunn et al. [6] obtained a few points in good agreement with the values computed from the dissociation curve [9]. Eq 3 yields values in close accord with the complex procedure of Thomas [10], and that of Ruiz et al. [8].

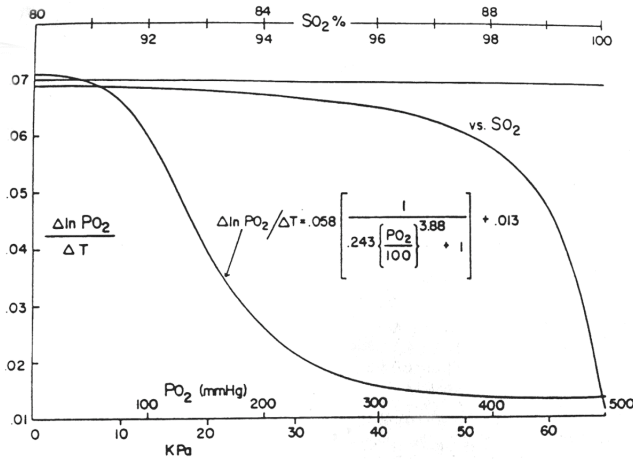


FIG. 2. Relationship of anaerobic temperature coefficient of whole blood PO_2 to the level of PO_2 (lower curve and lower abscissa, in both Torr and in kPa), or to saturation percentage (upper ordinates and curves). Nearly horizontal line at 0.07 is value from $80 < \text{sat}\% < 90$.

IV. Correction of PO_2 to $pH = 7.40$ (Bohr Effect)

Previously, the effect of respiratory variations of pH on PO_2 at constant saturation has been assumed to be independent of PO_2 . The Bohr effect is expressed as:

$$\Delta \log_{10} PO_2 / \Delta pH = \log[PO_{2\text{obs}} / PO_{2(7.4)}] / [pH_{\text{obs}} - 7.4] = -0.48$$

or using natural logarithms:

$$\Delta \ln PO_2 / \Delta pH = \ln[PO_{2\text{obs}} / PO_{2(7.4)}] / [pH_{\text{obs}} - 7.4] = -1.1 \quad (4)$$

In fact, the Bohr effect falls off at high saturation and increases at very low saturation, as illustrated in Fig. 3, using the data of Hlastala and Woodson [3]. An additional point Roughton and I obtained [7] at $S = 0.99$ is 0.78 (0.34 in \log_{10}) is not shown in Fig. 3. When CO_2 is the acid variable, about 20% of the Bohr effect is due to a CO_2 effect independent of pH . For this one may add a correction of $+0.003$ BE [9]. Defining $\Delta \ln PO_2 / \Delta pH$ as $\ln[PO_{2\text{obs}} / PO_{2(7.4)}] / [pH_{\text{obs}} - 7.4]$, an empiric estimate of the Bohr effect taking these data into account is then:

$$\Delta \ln PO_2 / \Delta pH = (PO_2 / 26.7)^{0.184} + 0.003 \text{BE} - 2.2 \quad (5)$$

V. Computing (A-V) PO_2 differences from O_2 content differences.

When blood delivers O_2 to tissue or absorbs O_2 in the lung, both dissolved O_2 and HbO_2 change. The partition ratio depends on the slope of the dissociation curve over which the blood moves. O_2 content (C , ml/dl) is computed from PO_2 , S and Hb (g/dl):

$$C = (1.34[Hb]S) + 0.0031PO_2 \quad (6)$$

It is necessary to iterate to minimum error using Eqs 1 and 2 to determine the change of PO_2 caused by a measured change of saturation or content, with or without changes of pH .

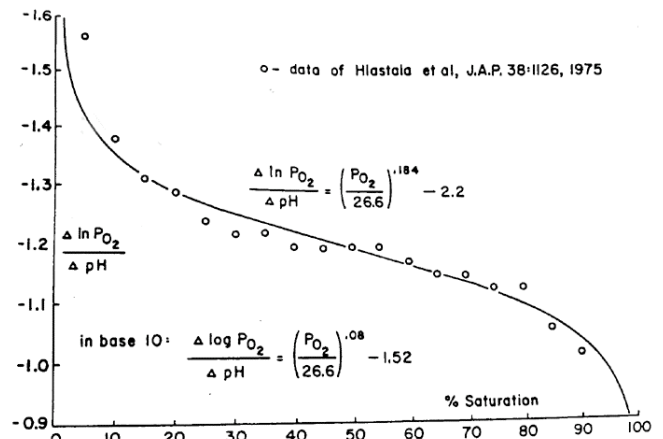


Fig. 3. The Bohr factor variation with saturation. Data of Hlastala were fitted with a Hill-type equation empirically. Assumptions: $BE=0$, 2,3-DPG is normal.

VI. Computation of P_{50}

This estimate can be done with blood with $0.2 < S < 0.8$. Measure PO_2 and pH at $37^\circ C$. Measure S spectrophotometrically. Use Eq. 2 to compute $PO_{2(7.4)}$ from measured S . Use Eq. 4 (or 5) to correct $PO_{2(\text{obs})}$ to $pH = 7.4$ from the measured PO_2 and pH . Then:

$$P_{50} = 26.7 PO_{2(\text{obs})} / PO_{2(7.4)} \quad (7)$$

This research was supported in part by National Institutes of Health Grants HL 06285, GM-15571-10, 5K06 HL-19412-16.
Received 1 May 1978; accepted 2 November 1978.

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Note: This version was revised in Nov. 1999, correcting errors and deleting some obsolete portions, and adding the inversion of equation 1 by R. Ellis [2], modified on 7/22/02 to clarify the sign problem when specifying ΔpH or ΔPO_2 . I further modified the text and equations for clarity in 2007. JWS